

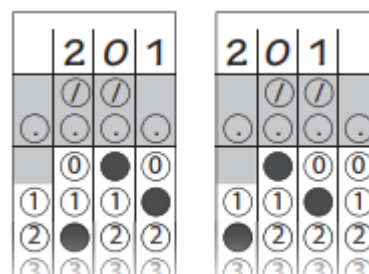
**DO NOT TURN TO THE NEXT PAGE**  
until your proctor tells you.

Please read the directions carefully.

- ◆ You have **100 minutes** for **40 Problems**.
- ◆ Mark your answers on your Answer Form with a pencil.
- ◆ Extra scratch paper is neither given nor allowed. You may use blank pages in the booklet as scratch paper.
- ◆ There are no penalties for incorrect answers. Answer as many problems as you can; return to the others in the time you have left for the test.
- ◆ Calculators are not permitted. Cell phones must be turned off completely and placed out of sight.
- ◆ The problems are divided into three categories, Part **A**, Part **B** and Part **C**, according to difficulty level. A correct answer for a Part A problem is worth 3 points, Part B is worth 5 points, and Part C is worth 7 points. Each problem is a multiple-choice problem except the last four problems in Part C.
- ◆ Problems 37-40, the last four problems of Part C, are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.

1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.

2. Mark no more than one circle in any column.
3. You may start your answers in any column, space permitting. Columns you don't use should be left blank, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable. For example: Answer: 201 – either position is correct.



4. No problem has a negative answer.

◆ **Notations in Geometry Problems:**

- $A$  : Point  $A$
- $\overleftrightarrow{AB}$  : Line through points  $A$  and  $B$
- $\overline{AB}$  : Line segment joining  $A$  and  $B$
- $AB$  : Length of the line segment  $\overline{AB}$ .
- $\angle ABC$  : Angle with the vertex point at  $B$
- $m\angle ABC$  : Measure of  $\angle ABC$
- $\perp$  : Perpendicular
- $//$  : Parallel

This page is intentionally left blank.

**Problem 1**

|          |
|----------|
| Algebra  |
| 3 Points |

If the function  $f$  satisfies  $f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$  for every positive real  $x$ , what is the value of  $f(2)$ ?

- A)  $\frac{1}{2}$
- B)  $\frac{3}{2}$
- C) 3
- D) 4
- E) 6

**Problem 2**

|               |
|---------------|
| Combinatorics |
| 3 Points      |

Start with 243. In each blank below, insert either  $\times 3$  or  $\div 9$  to create a true equation. How many different, true equations can be formed?

$$243 \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad = 1$$

- A) 5
- B) 12
- C) 35
- D) 128
- E) 256

**Problem 3**

Algebra

5 Points

A parabola with equation  $y = ax^2 + bx + c$  has vertex  $(h, k)$ . How many of the six quantities  $a, b, c, h, k$  and  $\Delta = b^2 - 4ac$  can be negative at the same time?

- A) At most 2
- B) At most 3
- C) At most 4
- D) At most 5
- E) All six

**Problem 4**

Number Theory

5 Points

An integer  $N$  has 10 positive divisors. If  $2N$  has 15 positive divisors and  $3N$  has 20 positive divisors, how many positive divisors does  $4N$  have?

- A) 15
- B) 20
- C) 24
- D) 28
- E) 32

Problem 5

|         |
|---------|
| Algebra |
|---------|

|          |
|----------|
| 5 Points |
|----------|

Find the number of integer solutions to the following equation  $(x^2 - 3x + 1)^{x+1} = 1$ .

A) 0

B) 1

C) 2

D) 3

E) 4

**Problem 6**

Number Theory

7 Points

What is the number of ordered pairs  $(x, y)$  of positive integers that satisfy the equation  $2x + 3y = 120$ ?

- A) 19
- B) 24
- C) 29
- D) 36
- E) None of the preceding

**Problem 7**

Geometry

7 Points

In triangle  $ABC$ , the medians  $\overline{BE}$  and  $\overline{CD}$  intersect at  $F$  and are perpendicular to each other. If  $BE = 18$  and  $CD = 24$ , find the length  $AF$ .

- A) 15
- B) 20
- C) 25
- D) 30
- E) None of the preceding

**Problem 8**

Algebra

7 Points

What is the maximum possible value of  $x + y$  for positive integers  $x$  and  $y$  that satisfies the following equation?

$$\log_2(\log_{2^x}(\log_{2^y} 2^{400})) = 0$$

- A) 53
- B) 102
- C) 201
- D) 400
- E) None of the preceding

**Problem 9**

Combinatorics

7 Points

Both FIZ and FIZZ are acceptable Scrabble words, though the version with two Z's is better known. How many six-letter strings formed only from the letters F, I, and Z will contain the word FIZ but not the word FIZZ?

- A) 64
- B) 72
- C) 80
- D) 96
- E) None of the preceding

Problem 10

|          |
|----------|
| Geometry |
| 7 Points |

On a semicircle with diameter  $\overline{AB}$ , two points  $C$  and  $D$  are taken such that  $BC = CD = 2$ . If  $AB = 6$ , find the value of  $6AD$ .

