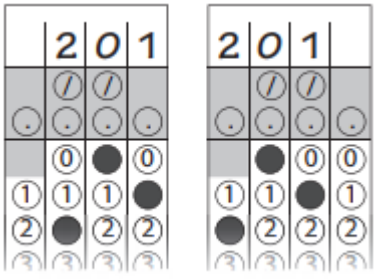
# MATHCON 2019 Finals

GRADE **11**

## DO NOT TURN TO THE NEXT PAGE until your proctor tells you.

## Please read the directions carefully.

- You have **100 minutes** for **40 Problems**.
- Mark your answers on your Answer Form with a pencil.
- Extra scratch paper is neither given nor allowed. You may use blank pages in the booklet as scratch paper.
- There are no penalties for incorrect answers. Answer as many problems as you can; return to the others in the time you have left for the test.
- Calculators are not permitted. Cell phones must be turned off completely and placed out of sight.
- The problems are divided into three categories, Part **A**, Part **B** and Part **C**, according to difficulty level. A correct answer for a Part A problem is worth 3 points, Part B is worth 5 points, and Part C is worth 7 points. Each problem is a multiple-choice problem except the last four problems in Part C.
- Problems 37-40, the last four problems of Part C, are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.
  1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.
  2. Mark no more than one circle in any column.
  3. You may start your answers in any column, space permitting. Columns you don't use should be left blank, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable.
     For example: Answer: 201 – either position is correct.




  4. No problem has a negative answer.

- **Notations in Geometry Problems:**

| | |
|---|---|
| $A$ | : Point $A$ |
| $\overleftrightarrow{AB}$ | : Line through points $A$ and $B$ |
| $\overline{AB}$ | : Line segment joining $A$ and $B$ |
| $AB$ | : Length of the line segment $\overline{AB}$. |
| $\angle ABC$ | : Angle with the vertex point at $B$ |
| $m\angle ABC$ | : Measure of $\angle ABC$ |
| $\perp$ | : Perpendicular |
| $//$ | : Parallel |

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**Problem 1**

| Algebra |
| --- |
| 3 Points |

If $\log_2 3 = a$ and $\log_3 5 = b$, what is the value of $\log_2 60$ in terms of $a$ and $b$?

A) $2b + a$

B) $a + b - 1$

C) $ab + a + 2$

D) $ab + 2$

E) None of the preceding

**Problem 2**

| Number Theory |
| --- |
| 3 Points |

How many integer values for $n$ make $n^{18/n}$ have an integer value?

A) 6

B) 7

C) 8

D) 9

E) 12

**Problem 3**

| Algebra |
| --- |
| 5 Points |

The sum of the first 20 terms of an arithmetic sequence which has first term 1 is equal to the sum of the first 10 terms of an arithmetic sequence which has first term 10. If positive integers $x$ and $y$ are the respective common differences of the sequences, what is the minimum value of $x + y$?

A) 35

B) 38

C) 40

D) 43

E) None of the preceding

**Problem 4**

| Number Theory |
| --- |
| 5 Points |

What is the remainder when $1^4 + 2^4 + 3^4 + \ldots + 2019^4$ is divided by 16?

A) 2

B) 4

C) 6

D) 8

E) 14

**Problem 5**

| Combinatorics |
| --- |
| 5 Points |

Let $x$, $y$, and $z$ be three numbers randomly picked with replacement from the set $\{1, 2, 3, 4, 5\}$, what is the probability that $xz + y$ is even number?

A) $\frac{2}{5}$

B) $\frac{23}{25}$

C) $\frac{39}{125}$

D) $\frac{64}{125}$

E) $\frac{59}{125}$

**Problem 6**

| Algebra |
| --- |
| 5 Points |

Suppose $x$ and $y$ are real numbers that satisfy $2x^2 - 3y = -\frac{17}{2}$ and $y^2 - 4x = 7$. Find the value of $x + y$.

A) $\frac{7}{2}$

B) $\frac{5}{4}$

C) $\frac{3}{2}$

D) $\frac{1}{4}$

E) None of the preceding

**Problem 7**

| Combinatorics |
| --- |
| 7 Points |

Suppose you have 98 identical real coins and 3 identical fake coins that look like real coins but are lighter in weight. By using only an equal-arm scale, what is the least number of weighings required to find 25 real coins?

A) 2

B) 3

C) 4

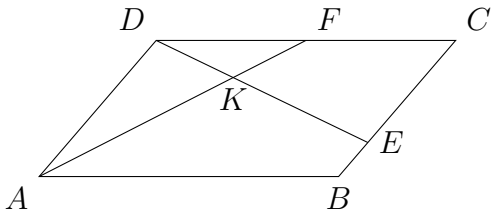
D) 5

E) None of the preceding

**Problem 8**

| Geometry |
| --- |
| 7 Points |

$ABCD$ is a parallelogram. $E$ and $F$ are two points on $\overline{BC}$ and $\overline{CD}$, respectively. If $CE = 3BE$, $CF = DF$, $\overline{DE} \cap \overline{AF} = \{K\}$ and $KF = 6$, find $AK$.




A) 10

B) 12

C) 14

D) 15

E) 16

**Problem 9**

| Algebra |
| --- |
| 7 Points |

Suppose $a$ and $b$ are real numbers such that $ab^2 = 1$ and $a^3 + 3b^3 = 4$. What is the product of all possible values of $a^3 + b^3$.

**Problem 10**

| Number Theory |
| --- |
| 7 Points |

An $EZ$ number is defined as any positive integer with the following properties:

- It has at least two digits.
- All its digits are the same.
- It has exactly 4 positive divisors

For example: $111 = 3 \times 37$ is an $EZ$ number. How many are less than $10^5$?