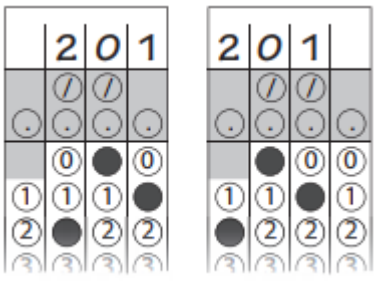
DO NOT TURN TO THE NEXT PAGE until your proctor tells you.

Please read the directions carefully.

- You have **100 minutes** for **40 Problems**.
- Mark your answers on your Answer Form with a pencil.
- Extra scratch paper is neither given nor allowed. You may use blank pages in the booklet as scratch paper.
- There are no penalties for incorrect answers. Answer as many problems as you can; return to the others in the time you have left for the test.
- Calculators are not permitted. Cell phones must be turned off completely and placed out of sight.
- The problems are divided into three categories, Part **A**, Part **B** and Part **C**, according to difficulty level. A correct answer for a Part A problem is worth 3 points, Part B is worth 5 points, and Part C is worth 7 points. Each problem is a multiple-choice problem except the last four problems in Part C.
- Problems 37-40, the last four problems of Part C, are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.

  1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.
  2. Mark no more than one circle in any column.
  3. You may start your answers in any column, space permitting. Columns you don't use should be left blank, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable.
     For example: Answer: 201 – either position is correct.



  4. No problem has a negative answer.

- **Notations in Geometry Problems:**

A	: Point A
$\overleftrightarrow{AB}$	: Line through points A and B
$\overline{AB}$	: Line segment joining A and B
AB	: Length of the line segment $\overline{AB}$.
$\angle ABC$	: Angle with the vertex point at B
$m\angle ABC$	: Measure of $\angle ABC$
$\perp$	: Perpendicular
$//$	: Parallel

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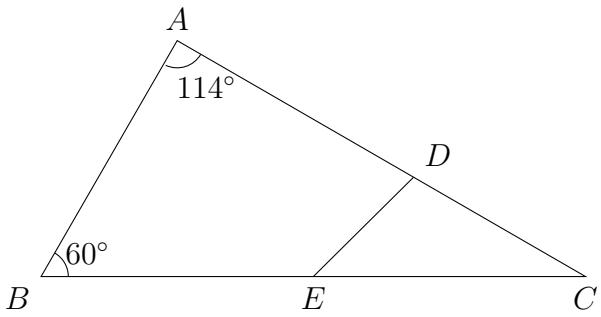
Problem 1

Geometry
3 Points

ABC is a triangle with$|AB| = |AD| = |BE|$.
$m\angle A = 114°$ and $m\angle B = 60°$. Find $m\angle EDC$.



A) $108°$

B) $112°$

C) $115°$

D) $120°$

E) None of the preceding

Problem 2

Algebra
3 Points

If $5^2 + 5^3 + 5^4 + \ldots + 5^{20} = M$, then what is the value of $5^2 + 5^3 + 5^4 + \ldots + 5^{18}$?

A) $M + 19$

B) $M - 5^{21}$

C) $\dfrac{M - 150}{25}$

D) $\dfrac{M - 36}{9}$

E) $\dfrac{M}{25}$

Problem 3

Number Theory
5 Points

Suppose a, b and c are distinct prime numbers such that $a - c = 8300$ and $a + b + c = 9704$. Find the value of $a + 2b$.

A) 9005

B) 9001

C) 8979

D) 8649

E) None of the preceding

Problem 4

Combinatorics
5 Points

In a regular octagon, all diagonals are drawn. If a diagonal is chosen at random, what is the probability that it is either one of the shortest or one of the longest?

A) $\frac{2}{5}$

B) $\frac{3}{5}$

C) $\frac{12}{25}$

D) $\frac{4}{5}$

E) None of the preceding

Problem 5

Number Theory
5 Points

As n ranges over all positive integers, how many distinct values can be found for the greatest common divisor of $6n + 15$ and $10n + 21$?

A) 2

B) 3

C) 4

D) 5

E) 6

Problem 6

Algebra
7 Points

If $\sqrt[3]{6\sqrt{3}+10} - \sqrt[3]{6\sqrt{3}-10} = a$, then find the value of $\dfrac{a^3}{10-3a}$.

A) 1

B) 2

C) 6

D) 20

E) None of the preceding

Problem 7

Number Theory
7 Points

If $x_1 < x_2 < \ldots < x_n$ are whole numbers, for some positive integer n, such that

$$2^{x_1} + 2^{x_2} + \ldots + 2^{x_n} = 160000.$$

Find the value of $x_1 + x_n$.

A) 36

B) 32

C) 28

D) 25

E) 24

Problem 8

Geometry
7 Points

Suppose $ABCD$ is a trapezoid such that $AB//CD$. $AD = 8$, $DC = 11$, $BC = 15$, and $m\angle C + m\angle D = 270$. Find the value of AB.

A) 26

B) 28

C) 30

D) 32

E) 34

Problem 9

Number Theory
7 Points

Suppose $n > 1$ is an integer. For each $k = 1, 2, \ldots, n$, let $d(k)$ be the number of digits in the value of k (e.g., $d(32) = 2, d(708) = 3, d(60093) = 5$, and so on). If

$$d(1) + d(2) + \cdots + d(n) = 2013,$$

find the digit sum of n (e.g., the digit sum of 65003 is $6 + 5 + 0 + 0 + 3 = 14$).

Problem 10

Geometry
7 Points

Suppose BD bisects $\angle ABC$ and $BD = 3\sqrt{5}$, $AB = 8$, and $DC = \dfrac{3}{2}$. Find $AD + BC$.