

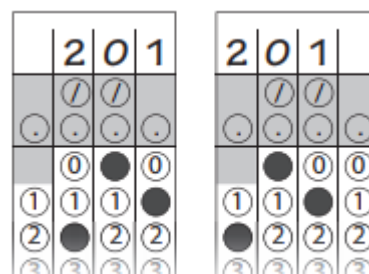
**DO NOT TURN TO THE NEXT PAGE**  
until your proctor tells you.

Please read the directions carefully.

- ◆ You have **100 minutes** for **40 Problems**.
- ◆ Mark your answers on your Answer Form with a pencil.
- ◆ Extra scratch paper is neither given nor allowed. You may use blank pages in the booklet as scratch paper.
- ◆ There are no penalties for incorrect answers. Answer as many problems as you can; return to the others in the time you have left for the test.
- ◆ Calculators are not permitted. Cell phones must be turned off completely and placed out of sight.
- ◆ The problems are divided into three categories, Part **A**, Part **B** and Part **C**, according to difficulty level. A correct answer for a Part A problem is worth 3 points, Part B is worth 5 points, and Part C is worth 7 points. Each problem is a multiple-choice problem except the last four problems in Part C.
- ◆ Problems 37-40, the last four problems of Part C, are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.

1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.

2. Mark no more than one circle in any column.
3. You may start your answers in any column, space permitting. Columns you don't use should be left blank, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable. For example: Answer: 201 – either position is correct.



4. No problem has a negative answer.

◆ **Notations in Geometry Problems:**

- $A$  : Point  $A$
- $\overleftrightarrow{AB}$  : Line through points  $A$  and  $B$
- $\overline{AB}$  : Line segment joining  $A$  and  $B$
- $AB$  : Length of the line segment  $\overline{AB}$ .
- $\angle ABC$  : Angle with the vertex point at  $B$
- $m\angle ABC$  : Measure of  $\angle ABC$
- $\perp$  : Perpendicular
- $//$  : Parallel

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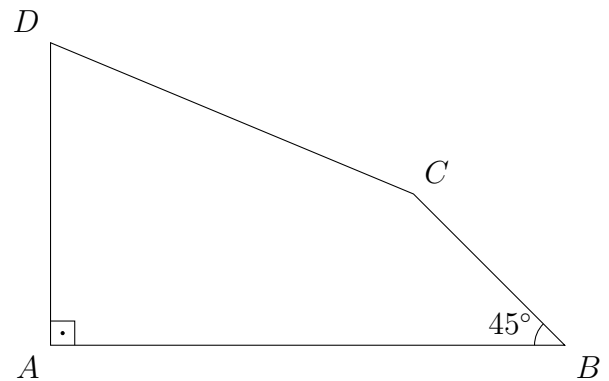
**Problem 1**

Geometry

3 Points

$ABCD$  is a quadrilateral with  $\overline{AB} \perp \overline{AD}$ .  
 $m\angle ABC = 45^\circ$ ,  $BC = 5\sqrt{2}$ ,  $AD = 10$  and  
 $AB = 17$ . Find  $DC$ .

- A)  $8\sqrt{2}$
- B) 12
- C)  $9\sqrt{2}$
- D) 13
- E) None of the preceding

**Problem 2**

Number Theory

3 Points

How many digits are in the number  $125^4 \cdot 64^2$ ?

- A) 10
- B) 11
- C) 12
- D) 13
- E) 14

**Problem 3**

Algebra

3 Points

What positive integer  $n$  satisfies the following equation?

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{16}$$

- A) 4
- B) 8
- C) 9
- D) 12
- E) 16

**Problem 4**

Number Theory

5 Points

Let  $N = 12345678910111213 \dots 394041$  be the 73-digit number obtained by writing the integers from 1 to 41 in order, one after the other. What is the remainder when  $N$  is divided by 9?

- A) 1
- B) 5
- C) 6
- D) 7
- E) 9

**Problem 5**

Geometry

5 Points

The sides of a triangle have lengths 7, 14, and  $c$ . For how many positive integer values of  $c$  is the triangle acute?

- A) 2
- B) 3
- C) 4
- D) 5
- E) 8

**Problem 6**

Number Theory

5 Points

The sum of three positive integers is 532. How many zeros, at most, does the product of those three numbers end with?

- A) 7
- B) 6
- C) 5
- D) 4
- E) 3

**Problem 7**

Combinatorics

5 Points

How many positive integers less than 2019 are multiples of 3 or 4 but not 5?

- A) 792
- B) 807
- C) 917
- D) 927
- E) None of the preceding

**Problem 8**

Algebra

7 Points

How many real number solutions does the equation  $5^x x^2 + 125 = 5^{x+2} + 5x^2$  have?

- A) 0
- B) 1
- C) 3
- D) 5
- E) Infinitely Many

**Problem 9**

Number Theory

7 Points

What is the minimum value of  $x + y + z + t$  where  $x, y, z$  and  $t$  are positive integers satisfying  $3^{8x} + 3^{5y} + 3^{12z} = 3^{19t}$ ?

**Problem 10**

Combinatorics

7 Points

In how many ways can a blank  $3 \times 3$  grid be filled with the integers from 1 to 9 so that squares containing consecutive integers are adjacent (i.e., have a common edge)? Hint: Of the three examples below, **A** and **B** satisfy the given conditions, while **C** does not because the squares containing 1 and 2 are not adjacent!

9	8	7
2	1	6
3	4	5

**A**

7	8	9
6	5	4
1	2	3

**B**

1	3	2
6	5	4
7	8	9

**C**