1. [Number Theory, 3 Points]

For positive integers p, r and s, and with $p < r < s$.
If $p + r = 7$ and $r + s = 10$, then what is the value of $\mathbf{p} \cdot \mathbf{r} \cdot \mathbf{s}$?

A) 84 B) 72 C) 48 D) 36 E) 24

2. [Combinatorics, 3 Points]

ab is a two-digit number where the ratio of a to b is 1:2.
How many such numbers are possible?

A) 4 B) 6 C) 8 D) 10 E) 12

3. [Algebra, 3 Points]

Given that for the non-zero real numbers,

$a \cdot b = b$

$b \cdot c = a$.

What is the value of $\mathbf{a} + \mathbf{b}$ in terms of c?

A) $\frac{c}{c-1}$ B) $\frac{c-1}{c}$ C) $\frac{c+1}{c}$ D) $\frac{c}{c+1}$ E) $\frac{c-1}{c+1}$

4. [Geometry, 3 Points]

In the Figure 1, vertex B and vertex D of the square ABCD are folded and they coincide at the midpoint O of the square, then the hexagon AEFCGH is obtained in the Figure 2.

What is the area of the hexagon AEFCGH if the area of square ABCD is 64 cm^2?

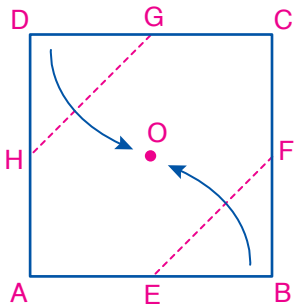


Figure 1

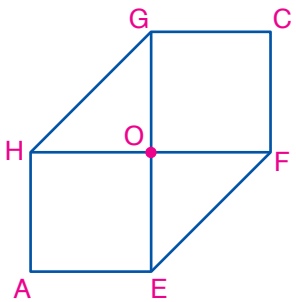


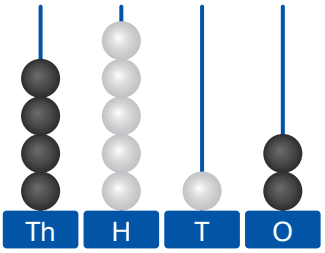
Figure 2

A) 24 cm^2 B) 32 cm^2 C) 40 cm^2 D) 48 cm^2 E) 56 cm^2

5. [Combinatorics, 5 Points]

We have 6 black and 6 white beads to show 4-digit numbers on a spike abacus.

- We will use only black beads in thousands and ones place, only white beads in tens and hundreds place.
- There must be at least one bead on each spike.
- We can use as many beads as we want.



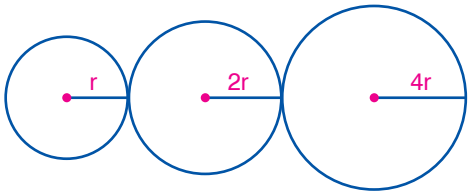
For example, the number in the figure is 4512.

Which of the following numbers **can not** be obtained if we used all the beads?

A) 3243 B) 2134 C) 4152 D) 1245 E) 5331

6. [Geometry, 5 Points]

The figure is the starting positions of three gears with radius r, 2r, and 4r which are connected to each other and rotating together.



Which of the following is the positions of the radii of the gears if the gear with the radius 4r is rotated 270° in the clockwise direction?

A) 

B) 

C) 

D) 

E) 

7. [Algebra, 5 Points]

$$x = \frac{8}{11}$$

$$y = \frac{80}{111}$$

$$z = \frac{800}{1111}$$

Which of the following is the order of x, y, and z from smallest to greatest?

A) $x < y < z$ B) $x < z < y$ C) $y < z < x$ D) $z < x < y$ E) $z < y < x$

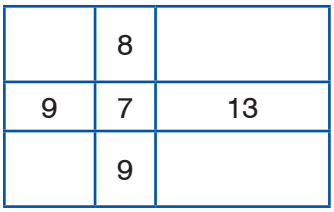
8. [Number Theory, 5 Points]

The difference of two numbers is 968.
When the larger number is divided by the smaller one, the quotient is 25 and the remainder is 6.
What is the smaller number?

A) 47 B) 43 C) 41 D) 39 E) 37

9. [Geometry, 7 Points]

The large rectangle on the right is divided into nine small rectangles.
The perimeters of five small rectangles given in the figure, in cm.
What is the perimeter of the large rectangle?



A) 32 cm B) 34 cm C) 35 cm D) 36 cm E) 40 cm

10. [Number Theory, 7 Points]

For positive integers x and y and three-digit number ABC,

$$ABC = 5x + 2 = 7y + 4.$$

What is $x + y$ for the smallest ABC that satisfies the given conditions above?

A) 34 B) 36 C) 38 D) 40 E) 42