

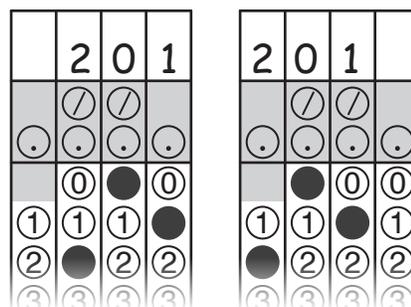
DO NOT TURN TO THE NEXT PAGE until your proctor tells you.

Please read the directions carefully.

- ▶ You have 100 minutes for 32 Problems.
- ▶ Mark your answers on your Answer Form with a pencil.
- ▶ Extra scratch paper is neither given nor allowed. You may use blank pages/spaces in the booklet as scratch paper.
- ▶ There are no penalties for incorrect answers. Answer as many problems as you can; go back and check your work and also go to questions you skip, before the time is over.
- ▶ Calculators are not permitted. Cell phones must be turned off completely and placed out of sight. MathCON problems are ALL done without a calculator.
- ▶ The problems are divided into three categories by difficulty levels:
 - 3 Points (Questions 1-8)
 - 5 Points (Questions 9-24)
 - 7 Points (Questions 25-32)
- ▶ Problems 29-32, the last four problems are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.
 1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.

2. Mark no more than one circle in any column.
3. You may start your answers in any column, space permitting. Columns you don't use should be left blanks, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable.

For example: Answer: 201 – either position is correct.



4. No problem has a negative answer.
- ▶ Notations in Geometry Problems:
 - A : Point A
 - \overleftrightarrow{AB} : Line through points A and B
 - \overline{AB} : Line segment joining A and B
 - AB : Length of the line segment AB
 - $\angle ABC$: Angle with the vertex point at B
 - $m\angle ABC$: Measure of angle ABC
 - \perp : Perpendicular
 - // : Parallel

3. [Number Theory, 3 Points]

Let X be a whole number.

- First multiply X by 9.
- Then add the digits of this product.
- This sum is called the 'core number of X '.

For example, $12 \rightarrow 12 \cdot 9 = 108 \rightarrow 1 + 0 + 8 = 9$, then the core number of 12 is 9.

Which of the following **cannot** be the core number of a four-digit number?

- A) 9 B) 18 C) 27 D) 36 E) 45

4. [Combinatorics, 3 Points]

There are six mailboxes numbered 1 to 6 at the entrance of an apartment.

- There is at least one letter in each box.
- The number of letters contained in each box is different from this box's number. For example, mailbox 5 does not contain five letters.
- The number of letters in the boxes can be equal, although they may differ.
- A total of five letters are in mailboxes 2 and 4.
- A total of seven letters are in mailboxes 4 and 5.



In total, how many letters are in mailboxes 2 and 5?

- A) 11 B) 10 C) 9 D) 8 E) 7

7. [Number Theory, 3 Points]

Using positive integers from 1 to n , sequence (a_n) is obtained by repeating each number squared times itself.

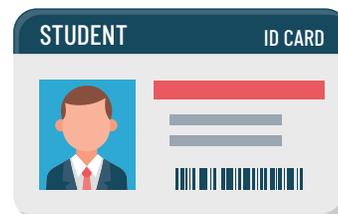
$$(a_n) = (1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, \dots, \underbrace{n, n, n, \dots, n, n, n}_{n^2 \text{ times}}, \dots)$$

What is the 205th term of (a_n) ?

- A) 6 B) 7 C) 8 D) 9 E) 10

9. [Algebra, 5 Points]

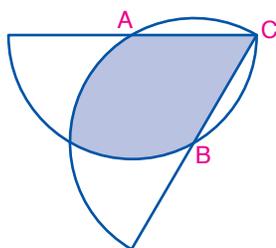
The photo occupies 20% of a student ID card. A 4 cm by 6 cm digital photo is reduced proportionally and fits into the space. What is the area of the ID card, if the perimeter of the photo is 8 cm?



- A) 19.2 cm^2 B) 19.6 cm^2 C) 20.4 cm^2 D) 21.1 cm^2 E) 22.4 cm^2

10. [Geometry, 5 Points]

\overline{AC} is the radius of the semicircle with center A, and \overline{BC} is the radius of the semicircle with center B. Point B is on the semicircle with center A, and point A is on the circle with center B.



$$AC = BC = 12 \text{ cm}$$

What is the area of the shaded region?

- A) $36\pi \text{ cm}^2$ B) $42\pi \text{ cm}^2$ C) $48\pi \text{ cm}^2$ D) $54\pi \text{ cm}^2$ E) $60\pi \text{ cm}^2$

12. [Combinatorics, 5 Points]

There are 18 different four-digit positive integers that can be made by arranging the digits 1, 2, 3, and 0. What is the sum of these 18 four-digit integers?

- A) 30884 B) 32664 C) 34884 D) 36884 E) 38664

