# DO NOT TURN TO THE NEXT PAGE
until your proctor tells you.

Please read the directions carefully.

▶ You have 100 minutes for 32 Problems.

▶ Mark your answers on your Answer Form with a pencil.

▶ Extra scratch paper is neither given nor allowed. You may use blank pages/spaces in the booklet as scratch paper.

▶ There are no penalties for incorrect answers. Answer as many problems as you can; go back and check your work and also go to questions you skip, before the time is over.

▶ Calculators are not permitted. Cell phones must be turned off completely and placed out of sight. MathCON problems are ALL done without a calculator.

▶ The problems are divided into three categories by difficulty levels:

3 Points (Questions 1-8)

5 Points (Questions 9-24)

7 Points (Questions 25-32)

▶ Problems 29-32, the last four problems are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.

1. Although not required, it is suggested that you write your answer from left to right in the boxes at tha top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.

2. Mark no more than one circle in any column.

3. You may start your answers in any column, space permitting. Columns you don't use should be left blanks, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable.

For example: Answer: 201 – either position is correct.
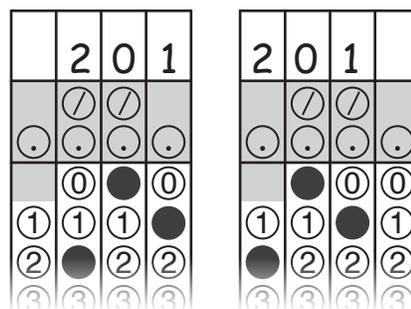
4. No problem has a negative answer.

▶ Notations in Geometry Problems:

A : Point A

$\overleftrightarrow{AB}$: Line through points A and B

$\overline{AB}$: Line segment joining A and B

AB : Length of the line segment AB

∠ABC : Angle with the vertex point at B

m∠ABC : Measure of angle ABC

⊥ : Perpendicular

// : Parallel

2. [Geometry, 3 Points]

Two identical cylindrical cheeses are cut vertically into two identical pieces, as shown in Figure 1 and Figure 2.

If the ratio of the area of the circular cross-section in Figure 2 to the area of the rectangular cross-section in Figure 1 is $\frac{2\pi}{9}$, what is the ratio of the height of the cheese to its radius initially?
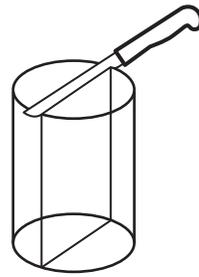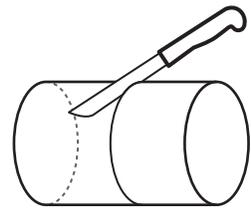
Figure 1   Figure 2

A) $\frac{7}{2}$ B) $\frac{7}{6}$ C) $\frac{9}{2}$ D) $\frac{9}{4}$ E) $\frac{3}{2}$

3. [Number Theory, 3 Points]

If a whole number is the product of two different prime numbers, then it is called a 'half-prime number'. For example, 15 is a half-prime number since $15 = 3 \cdot 5$.
Which of the following statements is/are true?

   I. There are five half-prime numbers between 20 and 30.
  II. The product of a prime number and a half-prime number is a half-prime number.
 III. Each half-prime number has only four positive divisors.

A) Only I   B) Only III   C) Only I and II   D) Only II and III   E) I, II, and III

5. [Algebra, 3 Points]

The heights of four trees, maple, birch, pine, and sequoia, form an arithmetic sequence in that order. Sequoia is 123 feet tall, and the average height of the four trees is 117 feet. How tall is maple?

PINE   SEQUOIA   BIRCH   MAPLE
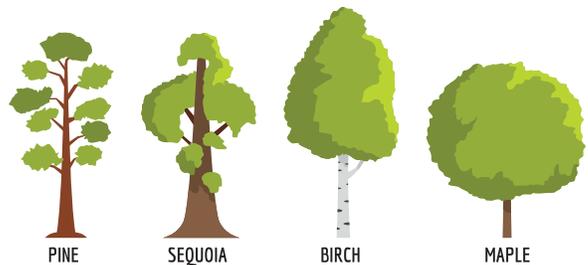
A) 111 feet   B) 113 feet   C) 115 feet   D) 119 feet   E) 121 feet

8. [Combinatorics, 3 Points]

In a tournament, five teams are participating: K, L, M, N, and T. The table shows Ander, Dior, and Kobe's predictions for which team will finish where. Which team finished second, if Ander, Dior, and Kobe each predicted three of the positions correctly?

	1	2	3	4	5
Ander	T	N	K	L	M
Dior	N	T	M	L	K
Kobe	T	M	N	L	K

A) K   B) L   C) M   D) N   E) T

11. [Number Theory, 5 Points]

Steps of a computer program are given below.

Step 1: $a = 1$ and $b = 1$.
Step 2: The new value of a equals the product of a and b.
Step 3: The new value of b equals one more than the previous value of b.
Step 4: If $a < 100$ then go to Step 2. Then follow Step 3, and Step 4.
        If $a > 100$ then stop. This value is the output.

What is the output of the program?

A) 720   B) 360   C) 240   D) 128   E) 120

12. [Combinatorics, 5 Points]

A six-sided number cube has the following numbers on its sides: 1, –2, 3, –4, 5, and –6. If Elijah throws this number cube four times, the sum of the four numbers he obtains **cannot** be equal to

A) –15   B) –16   C) –17   D) –18   E) –19